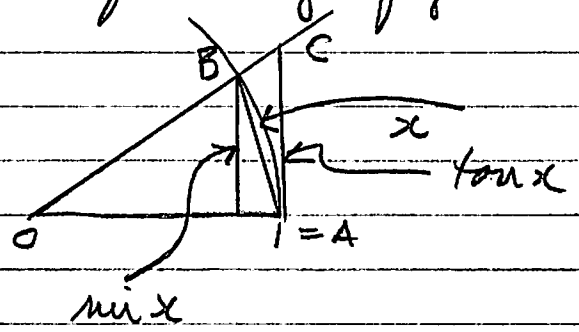
9/30/08

$$\frac{\sin x}{x} \longrightarrow 1 \quad \text{as } x \to 0$$

Consider the following figure:



$$\text{area } \Delta OAB < \text{area sector } OAB < \text{area } \Delta OAC$$

$$\tfrac{1}{2} \cdot 1 \cdot \sin x < \pi \cdot \frac{x}{2\pi} < \tfrac{1}{2} \cdot 1 \cdot \tan x \quad \text{if } 0 < x < \pi/2$$

Hence $0 < \sin x < x < \tan x$

$$\Rightarrow \quad 1 < \frac{x}{\sin x} < \cos x$$

$$\cos x \to 0 \quad \text{as } x \to 0$$

$$\therefore \quad \frac{x}{\sin x} \to 1 \quad \text{as } x \to 0.$$