Math 336 Sample Problems

One notebook sized page of notes will be allowed on the test. The test will cover up to section 2.6 in the text.

1. Suppose that v is the harmonic conjugate of u and u is the harmonic conjugate of v. Show that u and v must be constant.

2. Suppose $\sum_0^\infty |a_n|^2$ converges. Prove that $f(z) = \sum_0^\infty a_n z^n$ is analytic for $|z| < 1$. Compute $\lim_{r\to 1} \int_0^{2\pi} |f(re^{it})|^2 dt$.

3. Let a be a complex number and suppose $|a| < 1$. Let $f(z) = \frac{z-a}{1-\overline{a}z}$. Prove the following statments.

   (a) $|f(z)| < 1$, if $|z| < 1$.

   (b) $|f(z)| = 1$, if $|z| = 1$.

4. Let $z_j = e^{\frac{2\pi ij}{n}}$ denote the n roots of unity. Let $c_j = |1 - z_j|$ be the $n-1$ chord lengths from 1 to the points $z_j, j = 1, \ldots .n-1$. Prove that the product $c_1 \cdot c_2 \cdots c_{n-1} = n$. *Hint*: Consider $z^n - 1$.

5. Let $f(z) = x + i(x^2 - y^2)$. Find the points at which f is complex differentiable. Find the points at which f is complex analytic.

6. Find the Laurent series of the function $\frac{1}{z}$ in the annulus $D = \{z : 2 < |z-1| < \infty\}$.

7. Using the calculus of residues, compute

$$\int_{-\infty}^{+\infty} \frac{dx}{1+x^4}$$

8. Let $f(z) = \dfrac{p'(z)}{zp(z)}$, where $p(z) = \displaystyle\prod_{j=1}^{n}(z - z_j)$ and the z_j are distinct and different from 0. Find all the poles of f and compute the residues of f at these poles.

9. Let f be analytic within and on a simple closed curve Γ. Prove that $Re\left(\int_\Gamma \overline{f}(z)f'(z)dz\right) = 0$.

10. Compute $\displaystyle\int_{|z|=r} \frac{|dz|}{|z-a|^2}$, where $|a| \neq r$. Use the fact that on $\{|z| = r\}$, $|dz| = -ir\dfrac{dz}{z}$; and then use the Cauchy integral formula.

11. You will need to know the definitions of the following terms and statements of the following theorems.

    (a) Absolute Value (Modulus) and Argument of a complex number
    (b) $\lim_{z\to a} f(z)$
    (c) Continuity
    (d) Complex Derivative
    (e) Cauchy-Riemann equations
    (f) Harmonic Conjugate
    (g) Complex Analytic
    (h) Differentiability of Power Series
    (i) Complex Exponential Function
    (j) Complex Logarithm
    (k) Cauchy's Integral Theorem
    (l) Cauchy's Integral Formula

(m) Morera's Theorem
(n) Liouville's Theorem
(o) Isolated Singularities (types)
(p) Residues
(q) Residue Theorem
(r) Laurent Series

13. There may be homework problems or example problems from the text on the midterm.