Math 336 Sample Problems

One notebook sized page of notes will be allowed on the test. The test will cover up to §VIII.2 in the text (excluding those sections for which there was no homework).

1. p. 158, # 13.

2. Using Rouché's theorem, show that $z^5 + 5z^3 + z - 2$ has three roots in the set $\{z : |z| < 1\}$.

3. Suppose $f(z)$ is an entire function and $|f(z)| < 1 + |z|^{1/2}$. Prove that f is constant.

4. Let $D = \{z : |z| < 1\}$. Suppose f is analytic on an open set that includes the closure of D and suppose $|f(z)| < 1$ if $|z| = 1$. Prove that there is a unique $\zeta \in D$ such that $f(\zeta) = \zeta$.

5. Let u and v be harmonic on an open connected set W. Suppose that $u(z)v(z) = 0$ on an open subset of W. Prove that either u or v is identically 0 on W.

6. Let Ω be a bounded connected open set. Suppose $0 \in \Omega$ and that f is an analytic function on Ω such that if $z \in \Omega,\ f(z) \in \Omega$. Suppose also that $f(0) = 0,\ f'(0) = 1$. Prove that $f(z) = z$. Hint: use Cauchy's inequalities on the functions obtained by composing f with itself k times.

7. Let f be analytic on $D = \{|z| < 1\}$. Suppose f is $1 - 1$ on $D - \{0\}$. Prove that f is $1 - 1$ on D.

8. Suppose f is analytic in $\{0 < |z| < r\}$ for some $r > 0$. Suppose also that $|f(z)| < |z|^{-1+\epsilon}$ in $\{0 < |z| < \delta\}$, where $\epsilon > 0$. Prove that f has a removable singularity at 0.

9. Let $D = \{z : |z| < 1\}$. Let f be analytic and non-constant on W, and suppose $\overline{D} \subset W$. Suppose $|f|$ is constant on ∂D. Prove that f has at least one zero in D.

10. Prove that
$$\sum_{n=1}^{\infty} d(n) z^n = \sum_{n=1}^{\infty} \frac{z^n}{1 - z^n} \text{ for} |z| < 1,$$
where $d(n)$ is the number of divisors of n. Carefully consider convergence issues.

11. Review contour integration computations.

12. There may be homework problems, example problems from the text, or requests for statements of theorems on the midterm.