Lindelof Maximum Principle

Theorem 1. *Suppose u is bounded and harmonic in a bounded open connected set W. Suppose $\limsup_{z\to p\in\partial W} u(z) \le M$ except for a countable set $\{q_j\}$. Then $u(z) \le M$.*

Proof. This proof needs to be modified. Choose $\epsilon_j > 0$ so that $\sum_1^\infty \epsilon_j < \infty$. Let d be the diameter of W. Let $a \in W$ and let c be the distance from a to ∂W. Then

$$w(z) = \sum \epsilon_j \log \frac{|z - q_j|}{d}$$

converges uniformly on $|z - a| < d/2$. Hence $u(z) - w(z)$ is harmonic on W and we can apply our proof for the case of a finite number of exceptional points. □

Remark 1. *It's not obvious how to modify this for a set of measure* 0.