

Tangent

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How to map a vertical strip to the unit disk? To make things simple let's assume that the strip is $S = \{z : |\Re z| < \frac{\pi}{4}\}$. Multiply by i , exponentiate, then square: $z \rightarrow w = e^{2iz}$. This maps the strip to the right half plane. Now map to the unit disk via $w \rightarrow \frac{w-1}{w+1}$. Composing we get

$$z \rightarrow \frac{e^{2iz} - 1}{e^{2iz} + 1} = \frac{1}{i} \tan(z).$$

We might as well drop the $\frac{1}{i}$ since it's a rotation. Thus

$$z \rightarrow \tan(z)$$

maps the vertical strip S to the unit disk.