# Sample Problems

Math 464

One notebook sized page of notes will be allowed on the final. You may write on both sides of the page. Scientific calculators in which no formulas or text has been stored will be allowed. This set of problems only covers material since the midterm. The final will be comprehensive. There will be questions covering the first half and second half of the course. The exam is at **8:30 a.m. on Wednesday, December 16**.

1. Let $Q(f) = af(\frac{1}{6}) + bf(\frac{1}{2}) + cf(\frac{5}{6})$ be a quadrature formula on $[0, 1]$.
   a) If $Q$ is to have precision at least 2, what are the values of $a, b, c$?
   b) What is the exact precision of the resulting quadrature?

2. Let
$$g(x) = \begin{cases} x^3 & \text{if } 0 \le x \le 1 \\ a(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1 & \text{if } 1 \le x \le 2 \end{cases}$$
   a) Is $g(x)$ a cubic spline on $[0, 2]$? Justify your answer.
   b) Is $xg(x)$ a cubic spline on $[0, 2]$? Justify your answer.
   c) What should the value of $a$ be if $g$ is to be a <u>natural</u> cubic spline?

3. a) What points $x_0, x_1, x_2, x_3$ should be used in $[0, 1]$ to minimize the error bound in polynomial interpolation at four points? Express your answer using the symbols Hint: Use $\cos x = \sqrt{\frac{1 + \cos 2x}{2}}$.
   b) What is the largest value attained by $(x-x_0)(x-x_1)(x-x_2)(x-x_3)$ in $[0, 1]$, where $x_0, x_1, x_2, x_3$ are the points chosen in part a)?

   *Be careful* in this problem, $[0, 1]$ is <u>not</u> the interval $[-1, 1]$.

4. a) Using your calculator, solve $e^{-x} = x$ by means of Newton's method. Let $x_0 = 0$. Compute $x_1, x_2, x_3, x_4, x_5$.
   b) If you were able to compute in infinite precision, for which $n$ would $x_n$ have 100 correct digits?

5. a) Let $p(x) = 2x^4 - 3x^3 + 4x^2 - 2x + 1$. *Use nested multiplication* to evaluate $p(2)$. Show the multiplications.
   b) Use nested multiplication to evaluate $p'(2)$. Show the multiplications. *Do <u>not</u> take the derivative of $p(x)$ and then use the polynomial $p'(x)$ to evaluate $p'(2)$.*

6. Let $p_3(x)$ be the unique polynomial of degree at most 3 that interpolates $f(x) = 2^{x+2}$ at the points $(x_0, x_1, x_2, x_3) = (-2, -1, 1, 2)$. Notice that 0 is <u>not</u> one of the points.

   a) Form the divided difference table for $f$ at these points and write $p_3(x)$ in its Newton form.

   b) Give the best upper bound for $\|f - p_3\|_\infty$ on $[-2, 2]$.