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Math 124
Midterm II
November 22, 2011

| Problem 1 | 15 |  |
| ---: | :---: | :--- |
| Problem 2 | 10 |  |
| Problem 3 | 7 |  |
| Problem 4 | 8 |  |
| Problem 5 | 10 |  |
| Total: | $\mathbf{5 0}$ |  |

- In addition to this cover page, your exam should contain 5 problems on 5 pages.

At the start of the test, check that you have a complete exam!

- Unless otherwise instructed, show all your work. Answers with no supporting work, or obtained by guess-and-check, will result in little or no credit, even if correct.
- Indicate your final answer by placing a box around it.
- Unless otherwise instructed, please leave your answers in exact form instead of a decimal approximation. For instance, $\frac{\pi}{2}$ instead of 1.57 , and $\sqrt{5}$ instead of 2.236 . Simplify all you can.
- If you need more room, use the backs of pages, but indicate to the grader that you have done so.
- Raise your hand if you have any questions.

1 (15 points) Compute the derivatives of the following functions. No need to simplify your answers.
a) $f(x)=\sin (\sqrt{x \ln (x)})$
b) $g(x)=\arctan \left(e^{\pi x}+5\right)$
c) $y=x^{\left(2^{x}\right)}$

2 (10 points) Consider the curve implicitly defined by the equation:

$$
y^{2}=3 x+4 \cos (x y)
$$

a) ( 6 pts$)$ Compute $\frac{d y}{d x}$ in terms of $x$ and $y$.
b) (4 pts) Find the tangent line equations at the y-intercepts of this curve.

3 (7 points) A curve has parametric equations

$$
\begin{aligned}
& x=3 t^{2}+2 \\
& y=4 t^{3}+2
\end{aligned}
$$

Find the equation of the tangent line that passes through the point $(2,0)$.

4 ( 8 pts ) A prison yard is swept each night by a rotating beam of light, which rotates clockwise at a constant rate of 1 revolution per minute. Joe the inmate is trying to escape from this prison tonight.
To do so, he needs to run along a prison wall, while staying right behind the beam of light (the rest of his escape plan does not involve calculus, so it's not relevant). The source of light is 40 feet from this wall. How fast must Joe run along the wall, in feet per minute, to keep behind the beam of light, when he is at a distance of 50 feet from the light?

$5(10 \mathrm{pts})$ Let $f(x)=\left(\sqrt[3]{x^{2}}-2\right)^{6}$
a) Determine the maximum value of this function on the interval $[-1,1]$.
b) Use a tangent line approximation to estimate the value of $f(0.95)$. Show all work.

