$\boxed{1}$ (15 points) Compute the derivatives of the following functions. No need to simplify your answers.

a) $f(x) = \sin\left(\sqrt{x\ln(x)}\right)$

$$f'(x) = \cos\left(\sqrt{x\ln(x)}\right)\frac{1}{2\sqrt{x\ln(x)}}(\ln(x)+1)$$

b) $g(x) = \arctan(e^{\pi x}+5)$

$$g'(x) = \frac{1}{1+(e^{\pi x}+5)^2}(e^{\pi x})(\pi)$$

c) $y = x^{(2^x)}$

$$\ln(y) = 2^x\ln(x)$$

$$\frac{1}{y}y' = (2^x)\ln(2)\ln(x) + 2^x\frac{1}{x}$$

$$y' = \boxed{x^{2^x}\left[(2^x)\ln(2)\ln(x) + \frac{2^x}{x}\right]}$$

$\boxed{2}$ (10 points) Consider the curve implicitly defined by the equation: $y^2 = 3x + 4\cos(xy)$

a) (6 pts) Compute $\frac{dy}{dx}$ in terms of x and y.

$$2y\,\frac{dy}{dx} = 3 - 4\sin(xy)(1y + x\frac{dy}{dx})$$

$$\boxed{\frac{dy}{dx} = \frac{3-4y\sin(xy)}{2y+4x\sin(xy)}}$$

b) (4 pts) Find the tangent line equations at the y-intercepts of this curve.

The y-intercepts are the points on the curve where $x = 0$, so that: $y^2 = 3(0) + 4\cos(0y) = 4$ => $y = \pm 2$. The y-intercepts are $(0,-2)$ and $(0,2)$.

$\left.\frac{dy}{dx}\right|_{(0,-2)} = \frac{3}{2(-2)} = -\frac{3}{4}$, so the tangent line at $(0,-2)$ is $\boxed{y = -\frac{3}{4}x - 2}$

$\left.\frac{dy}{dx}\right|_{(0,2)} = \frac{3}{2(2)} = \frac{3}{4}$, so the tangent line at $(0,2)$ is $\boxed{y = \frac{3}{4}x + 2}$

$\boxed{3}$ (7 points) A curve has parametric equations

$$x = 3t^2 + 2$$
$$y = 4t^3 + 2$$

Find the equation of the tangent line that passes through the point (2,0).

Let $t = a$ be the parameter value corresponding to the point of tangency.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{12t^2}{6t} = 2t \text{ if } t \neq 0 \ (DNE \text{ at } t = 0)$$

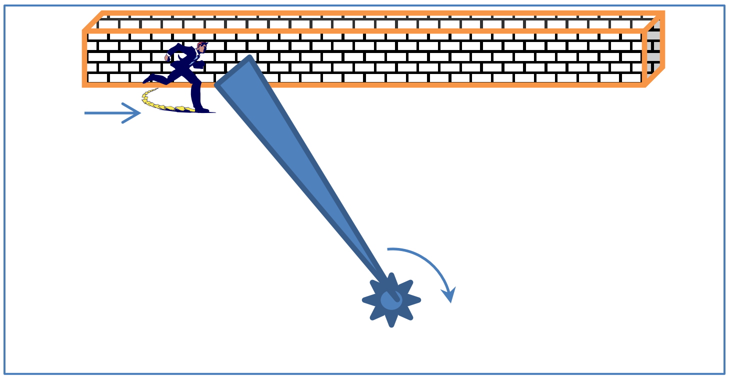
Writing the slope of the tangent line at $\big(x(a), y(a)\big)$ as the derivative at that point and as the rise over the run from that point to (2,0), we get:

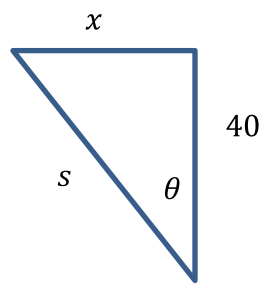
$$\text{slope} = 2a = \frac{y(a) - 0}{x(a) - 2} \ => 2a = \frac{4a^3 + 2}{3a^2} \ => 6a^3 = 4a^3 + 2 => 2a^3 = 2 => a^3 = 1 => a = 1$$

The point of tangency is: $(x(1), y(1)) = (5, 6)$, and the slope is 2.

The equation for the tangent line is $y = 2(x - 5) + 6$, i.e. $\boxed{y = 2x - 4}$.

$\boxed{4}$ (8 pts) A prison yard is swept each night by a rotating beam of light, which rotates clockwise at a constant rate of 1 revolution per minute. Joe the inmate is trying to escape from this prison tonight. To do so, he needs to run along a prison wall, while staying right behind the beam of light (the rest of his escape plan does not involve calculus, so it's not relevant). The source of light is 40 feet from this wall. How fast must Joe run along the wall, in feet per minute, to keep behind the beam of light, when he is at a distance of 60 feet from the light?





Know: $\frac{d\theta}{dt} = 1\frac{rev}{min} = 2\pi\frac{rad}{min}$

Want: $\frac{dx}{dt}$ when $s = 60\ ft.$

A relationship between x and θ (see the sketched right triangle) is: $\tan(\theta) = \frac{x}{40}$.

Differentiating with respect to time: $\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{40}\frac{dx}{dt}$

When $s = 50$, we can compute $\sec(\theta) = \frac{hypothenuse}{adjacent} = \frac{60}{40} = \frac{3}{2}$. Substituting: $\left(\frac{3}{2}\right)^2 2\pi = \frac{1}{40}\frac{dx}{dt}$

So $\frac{dx}{dt} = \boxed{180\,\pi \text{ ft/min}}$ $(\cong 565.49)$

$\boxed{5}$ (10 pts) Let $f(x) = \left(\sqrt[3]{x^2} - 2\right)^6$

a) Determine the maximum value of this function on the interval [-1, 1].

$f'(x) = 6\left(\sqrt[3]{x^2} - 2\right)^5\left(\frac{2}{3}\right)x^{-\frac{1}{3}}$

Critical points: $f'(x)\ DNE$ at $x = 0$ (in interval $[-1,1]$), and $f'(x) = 0\ at\ x = \pm 2\sqrt{2}$ (outside of interval)

Evaluate f at endpoints and at x=0: $f(-1) = f(1) = \left(\sqrt[3]{1} - 2\right)^6 = 1$ & $f(0) = (-2)^6 = 64$

$\boxed{\text{Maximum value of } f(x) \text{ is } 64}$, which is attained when $x = 0$.

b) Use a tangent line approximation to estimate the value of $f(0.95)$. Show all work.

Use the linearization of f at 1: $L(x) = f'(1)(x-1) + f(1) = -4(x-1) + 1$.

$f(0.95) \cong -4(0.95 - 1) + 1 = \boxed{1.2}$