University of Washington
Complex Analysis - Math 535
S. Rohde Winter 2018

Exercise Set 2

Problem 1: Prove the result stated in class: If $f_n : D \to \mathbb{C}$ are analytic, and if $\sum_n |f_n - 1|$ converges locally uniformly and absolutely in D, then the infinite product $\prod_n f_n$ converges absolutely and locally uniformly, the limit f is analytic and vanishes precisely at the roots of the f_n (counting multiplicities), and the series of logarithmic derivatives $\sum_n f_n'/f_n$ converges locally uniformly to f'/f.

Problem 2: Prove the estimate

$$|1 - E_p(z)| \leq |z|^{p+1}$$

for all integers $p \geq 0$ and all $|z| \leq 1$.

Problem 3. Let f be analytic and bounded in the strip $\{1/2 < \operatorname{Re} z < 2\}$, assume $f(1) = 1$, and assume that

$$f(z+1) = zf(z)$$

for $1/2 < \operatorname{Re} z < 1$. Prove that $f(z) \equiv \Gamma(z)$. Hint: Continue $f - \Gamma$ to $\mathbb{C}$.

Problem 4. Do problems 2 and 3 in (Gamelin, Complex Analysis, XIV.1, p. 364)

Problem 5. Do problem 2.8 of Schlag, A course in Complex Analysis and Riemann surfaces

Problem 6. Find a conformal map from the "inside" of a parabola $\{x+iy : y > x^2\}$ to the unit disc.

Due date : Monday, January 29, before class.