

Exercise Set 5

Do problems 3.10, 3.13a), 3.14 of Schlag, A course in Complex Analysis and Riemann surfaces, and the following problems:

Problem 4. Let $u \in C^2(G)$ (continuous second order derivatives). Show that u is subharmonic if and only if $\Delta u \geq 0$ in G . Hints: To prove one direction, show first that $u(z) + \epsilon x^2$ is subharmonic. For the other direction, show that the assumption $\Delta u(z_0) < 0$ would contradict the conclusion of problem 3.6 (iv) of Schlag's book.

Problem 5. Let $f : \mathbb{D} \rightarrow \mathbb{D} \setminus \{0\}$ be analytic. Show that

$$|f(z)| \geq |f(0)|^{\frac{1+|z|}{1-|z|}}$$

for $z \in \mathbb{D}$.

Due date : Monday, March 5, before class.