Lecturer: Thomas Rothvoss — Due date: Friday, Oct 13, 2023, 11pm, on GradeScope

Problem Set 2

514 - Networks and Combinatorial Optimization

Autumn 2023

Exercise 2.1 (10 pts)
Let $(X,\mathcal{I})$ be a pair with X finite and $\mathcal{I} \subseteq 2^X$. Consider the following properties:

(i) $\emptyset \in \mathcal{I}$

(ii) If $Y \in \mathcal{I}$ and $Z \subseteq Y$, then $Z \in \mathcal{I}$.

(iii) If $Y, Z \in \mathcal{I}$ and $|Y| < |Z|$, then $Y \cup \{x\} \in \mathcal{I}$ for some $x \in Z \setminus Y$.

(iv) For any $Y \subseteq X$, any two bases of Y have the same cardinality.

Assume that (i)+(ii) hold. Prove that (iii) and (iv) are equivalent.

Exercise 2.2 (10 pts)
Let $M = (X,\mathcal{I})$ be a matroid. An inclusion-wise minimal dependent set $Y \subseteq X$ is called a *circuit*. Two elements $x, y \in X$ are called *parallel* if $\{x,y\}$ is a circuit. Show that if x and y are parallel and $Y \in \mathcal{I}$ with $x \in Y$, then $(Y \setminus \{x\}) \cup \{y\} \in \mathcal{I}$.

Exercise 3.1 (10 pts)
Let $C \subseteq \mathbb{R}^n$. Prove that C is a closed convex set if and only if there is a collection $\mathcal{F}$ of closed affine halfspaces so that $C = \bigcap_{H \in \mathcal{F}} H$.

Remark. All exercises are taken from A. Schrijver's lecture notes.