Lecturer: Thomas Rothvoss | Due date: Friday, Nov 10, 2023, 11pm, on GradeScope

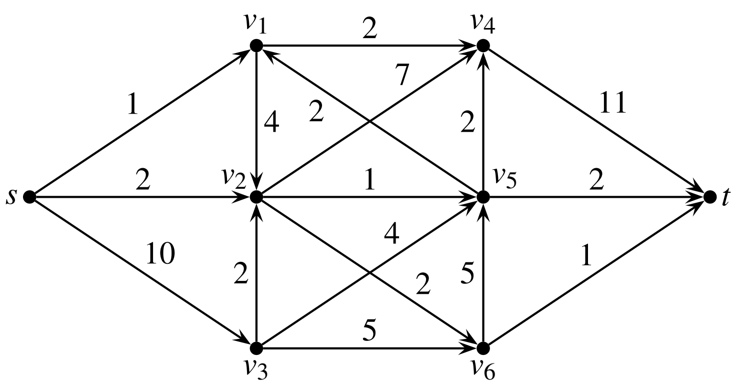
Problem Set 6

514 - Networks and Combinatorial Optimization

Autumn 2023

Exercise 6.1 (10pts)
Let $D = (V,A)$ be a directed graph and let $s,t \in V$. Let $f : A \to \mathbb{R}_{\geq 0}$ be an s-t flow of value β. Show that there exists an s-t flow $f' : A \to \mathbb{Z}_{\geq 0}$ of value $\lceil \beta \rceil$ so that $\lfloor f(a) \rfloor \leq f'(a) \leq \lceil f(a) \rceil$ for every $a \in A$.

Exercise 6.2 (10pts)
In the following graph $D = (V,A)$ (edges labelled with capacities $u(a)$), compute a maximum s-t flow under u and a minimum s-t cut $\delta^{out}(U)$. What are their values? It suffices to state the final outcomes.



Exercise 6.3 (10pts)
Let $D = (V,A)$ be a directed graph with two distinguished nodes $s,t \in V$. A set U is called an s-t *vertex cut* if $U \subseteq V \setminus \{s,t\}$ and every s-t path intersects U. A collection of s-t paths $P_1, \ldots, P_N$ is called *internally vertex disjoint* if they have no nodes in common other than s and t. Prove the following using the MaxFlow=MinCut Theorem: *Let $D = (V,A)$ be a directed graph with $s,t \in V$ so that $(s,t) \notin A$. Then the maximum number of internally vertex-disjoint s-t paths equals the minimum $|U|$ where U is an s-t vertex cut.*
Hint: Create an auxiliary graph and apply the MaxFlow=MinCut Theorem there!

Remark. Two exercises are taken from A. Schrijver's lecture notes.