Lecturer: Thomas Rothvoss Due date: Friday, Jan 13, 2023, 11pm, on GradeScope

Problem Set 1

CSE 599S - Lattices

Winter 2023

Exercise 1.1 (10pts)
Let $\Lambda = \Lambda(\boldsymbol{B})$ with $\boldsymbol{B} \in \mathbb{R}^{n\times n}$ be a full rank lattice. Show that for any $\varepsilon > 0$ there is a radius $R_0 := R_0(\varepsilon, n, \boldsymbol{B})$ so that for $R \geq R_0$ one has

$$(1-\varepsilon) \cdot \frac{\mathrm{Vol}_n(R \cdot B_2^n)}{\det(\Lambda)} \leq |RB_2^n \cap \Lambda| \leq (1+\varepsilon) \cdot \frac{\mathrm{Vol}_n(R \cdot B_2^n)}{\det(\Lambda)}$$

Exercise 1.2 (slightly adjusted; 10pts)
Let $K \subseteq \mathbb{R}^n$ be a symmetric convex set with $\mathrm{Vol}_n(K) > k \cdot 2^n$ for some $k \in \mathbb{N}$.

a) Show that $|K \cap \mathbb{Z}^n| \geq k$.

b) Is the following claim true? Explain!
For any $k \in \{1, \ldots, n\}$ there is a value $f(k,n)$ so that for any symmetric convex body K with $Vol_n(K) > f(k,n) \cdot 2^n$, the set $K \cap \mathbb{Z}^n$ contains k linearly independent vectors.