

Problem Set 7
CSE 599S - Lattices
Winter 2023

Exercise 4.1 (10pts)

Prove that in any full-rank lattice $\Lambda \subseteq \mathbb{R}^n$ one has $\mu(\Lambda) \leq n \cdot \lambda_n(\Lambda)$.

Extra point: Prove that even $\mu(\Lambda) \leq O(\sqrt{n}) \cdot \lambda_n(\Lambda)$.

Exercise 4.3 (10pts)

Prove the following statement: For any symmetric convex body $K \subseteq \mathbb{R}^n$ and any full rank lattice $\Lambda \subseteq \mathbb{R}^n$ one has $\lambda_1(K, \Lambda) \cdot \lambda_1(K^\circ, \Lambda^*) \leq Cn$ where $C > 0$ is a universal constant.

Hint: You may use the following deep result of Blaschke-Santaló-Bourgain-Milman without a proof: For any symmetric convex body $K \subseteq \mathbb{R}^n$ one has

$$C_1^n \leq \frac{\text{Vol}_n(K) \cdot \text{Vol}_n(K^\circ)}{\text{Vol}_n(B_2^n)^2} \leq 1$$

for some universal constant $C_1 > 0$.